SUMMIT MATH

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ALGEBRA 2

second edition

4 RATIONAL EXPRESSIONS & EQUATIONS

CONTENTS

Section 2

USING A DISGUISED FORM OF 1

If the previous addition scenario is easy for you, it is likely because you know how to change numbers by multiplying by a disguised form of 1. Since $\frac{2}{3}\cdot\frac{4}{4}=\frac{8}{12}$ it follows that $\frac{2}{3}$ has the same value as $\frac{8}{12}$. When $\frac{2}{3}$ is rewritten as $\frac{2}{3}\cdot\frac{4}{4}$ or $\frac{8}{12}$, it is easier to add it to $\frac{7}{12}$.

6. What disguised form of 1 is multiplied by $\frac{2}{3}$ to make it become $\frac{10}{15}$?

7. What disguised form of 1 is missing in each equation below?

a. $\frac{2}{3}\cdot\underline{\qquad}=\frac{18}{27}$ b. $\frac{2}{3}\cdot\underline{\qquad}=\frac{2x}{3x}$ c. $\frac{5y}{7}\cdot\underline{\qquad}=\frac{10y^2}{14y}$

8. What disguised form of 1 is missing in each equation below?

a. $5\cdot\underline{\qquad}=\frac{5x}{x}$ b. $\frac{1}{2}\cdot\underline{\qquad\qquad}=\frac{x-2}{2x-4}$ c. $\frac{3}{4}\cdot\underline{\qquad\qquad}=\frac{6x+15}{8x+20}$

9. What disguised form of 1 is missing in each equation below?

a. $\frac{x+2}{2x}\cdot\underline{\qquad\qquad}=\frac{x^2+7x+10}{2x^2+10x}$ b. $\frac{2x-1}{3x+2}\cdot\underline{\qquad\qquad}=\frac{6x^2-7x+2}{9x^2-4}$

10. In each equation below, a fraction is multiplied by a disguised form of 1 to create another fraction. Fill in the missing fraction to complete each equation.

a. $\frac{3x}{x-2}\cdot\frac{x+1}{x+1}=\underline{\qquad\qquad}$ b. $\frac{x-4}{x-3}\cdot\underline{\qquad\qquad}=\frac{x^2-12x+32}{x^2-11x+24}$

11. This book will show you how to use what you know about fractions to become familiar with fractions that contain variable expressions. These fractions are called rational expressions. A rational expression is a fraction that contains polynomials in the numerator and the denominator. Rewrite this definition below.

Section 3

SIMPLIFYING RATIONAL EXPRESSIONS

In previous scenarios, you multiplied by 1 to change the form of an expression. You can also change an expression by reversing this process.

12. Consider the fraction $\frac{9}{15}$. When you rewrite 9 and 15 in their factored forms, the fraction becomes $\frac{3\cdot3}{5\cdot3}$. When this is rewritten as $\frac{3}{5}\cdot\frac{3}{3}$, a disguised form of 1 can now be seen hiding in the fraction $\frac{9}{15}$. If this disguised form of 1 is removed (or canceled out, as it is often said), it becomes clear that $\frac{9}{15}$ is equivalent to the fraction _______.

13. Since $\frac{3}{5}$ contains "simpler" numbers than $\frac{9}{15}$, it is called the _________________ form of $\frac{9}{15}$.

14. Simplify each fraction below.

a. $\frac{8}{12}$ b. $\frac{-10}{12}$ c. $\frac{3+4}{6+8}$ d. $\frac{10-2}{5+2}$

15. Simplify each fraction.

a. $\frac{x}{x}$ b. $\frac{4x}{6x}$ c. $\frac{x+1}{1+x}$ d. $\frac{y-7}{y-7}$

16. Simplify each fraction by removing (or canceling out) the disguised form of 1.

a. $\frac{2\cdot10}{3\cdot10}$ b. $\frac{6\cdot5}{5\cdot3}$ c. $\frac{(x+5)(x-5)}{(x+5)}$ d. $\frac{(x+4)(x-2)}{x(x-2)}$

17. Simplify each fraction by canceling out (or removing) the disguised form of 1.

a. $\frac{12x}{20}$ b. $\frac{28}{7x}$ c. $\frac{x+1}{3x+3}$ d. $\frac{x^2-25}{x+5}$

Section 6

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

When you compare multiplying and dividing fractions, division is almost identical to multiplication. When a scenario requires dividing by a fraction, you should change the division to multiplying by the reciprocal. Similarly, when you compare adding and subtracting fractions, subtraction is almost identical to addition. The following scenarios will guide you through this.

56. First, it will be useful to review how to add and subtract fractions.

a. $\frac{2}{5}+\frac{3}{10}$

b. $\frac{3}{4}-\frac{1}{6}$

57. In the previous scenario, two fractions cannot be added unless their denominators are the same. The fractions need a common denominator. Find the smallest positive value that could be the common denominator for each pair of fractions.

a. $\frac{1}{3}$ and $\frac{1}{2}$

b. $\frac{5}{6}$ and $\frac{4}{15}$

c. $\frac{3}{5}$ and $\frac{2}{x}$

58. Simplify $\frac{5}{6}+\frac{2}{15}$.

59. Combine the following fractions. Simplify the resulting fraction as much as possible.

a. $\frac{2}{x}+\frac{3}{x}$

b. $\frac{x}{x+3}+\frac{3}{x+3}$

60. Combine the following fractions. Simplify the resulting fraction as much as possible.

a. $\frac{3}{x+1}-\frac{5}{x+1}$

b. $\frac{2x+1}{x-1}-\frac{x+2}{x-1}$

61. How can you make the expression $\frac{2}{x}+\frac{3}{4x}$ become a single fraction? Explain this with words and then use what you have written to write the expression as a single fraction.

Section 7
EQUATIONS WITH RATIONAL EXPRESSIONS

83. Simplify each expression as much as possible.

a. $\frac{2}{5}\cdot 5$ b. $\frac{x+2}{6}\cdot 6$ c. $\frac{y+3}{x}\cdot 2x$ d. $\frac{g-4}{3x^2}\cdot 6x^3$

84. None of the previous expressions contain fractions after they are simplified completely. Why does this happen?

85. Simplify each expression as much as possible.

a. $\frac{7}{x+5}\cdot(x+5)$ b. $\frac{x+2}{x}\cdot x(x+7)$ c. $\frac{3}{x-2}\cdot(x+2)(x-2)$

86. Fill in the blank to complete the equation.

a. $\frac{11}{x-4}\cdot$ ______ $=11$ b. $\frac{x-1}{x}\cdot$ ________ $=(x-1)(x-3)$

87. Use the distributive property to simplify each expression as much as possible.

a. $5x\cdot\left(\frac{5}{x}+\frac{3}{5}\right)$ b. $6x^2\cdot\left(\frac{2}{3x^2}+\frac{x+1}{2x}\right)$

88. Use the distributive property to simplify each expression as much as possible.

$$(x+4)(x+3)\cdot\left(\frac{3}{x+4}-\frac{7}{x+3}\right)$$

89. Solve each equation using standard equation-solving strategies. Think about how you solve them, because you will apply these strategies to solve more complicated equations.

a. $\frac{x}{6}=1.37$

b. $\frac{y+3.5}{2.4}=3.7$

c. $\frac{f}{5.2}-3.1=7.7$

Each of the previous three equations had a fraction with a number in the denominator. You can "move" this number by multiplying both sides of the equation by that number. This is sometimes called "clearing the fractions," even though it is really a method for clearing out the *denominators*. Just as you would clear out the road if fallen branches made it difficult for you to drive through, you can also clear out the denominators if they make it difficult for you to solve an equation.

90. This strategy is also useful for solving equations with more than one fraction.

a. To clear the one fraction in $\frac{x}{-2}=7$, multiply both sides of the equation by ____.

b. To clear the two fractions in $\frac{x}{5}=\frac{5}{2}$, multiply both sides of the equation by ____.

c. To clear the two fractions in $\frac{1}{4}=\frac{2x-1}{6}$, multiply both sides of the equation by ____.

Section 10
ANSWER KEY

1.	2
2.	20
3.	$\frac{3}{7}$ of a chocolate bar
4.	$\frac{4}{4}$, or 1 cup of water
5.	Yes; 5 quarters
6.	$\frac{5}{5}$
7.	a. $\frac{9}{9}$ b. $\frac{x}{x}$ c. $\frac{2y}{2y}$
8.	a. $\frac{x}{x}$ b. $\frac{x-2}{x-2}$ c. $\frac{2x+5}{2x+5}$
9.	a. $\frac{x+5}{x+5}$ b. $\frac{3x-2}{3x-2}$
10.	a. $\frac{3x^2+3x}{x^2-x-2}$ b. $\frac{x-8}{x-8}$
11.	A rational expression is a fraction that contains polynomials in the numerator and the denominator.
12.	$\frac{3}{5}$
13.	simplified
14.	a. $\frac{2}{3}$ b. $-\frac{5}{6}$ c. $\frac{7}{14}\rightarrow\frac{1}{2}$ d. $\frac{8}{7}$
15.	a. 1 b. $\frac{2}{3}$ c. 1 d. 1
16.	a. $\frac{2}{3}$ b. 2 c. $x - 5$ d. $\frac{x+4}{x}$
17.	a. $\frac{4\cdot 3x}{4\cdot 5}\rightarrow\frac{3x}{5}$ b. $\frac{7\cdot 4}{7\cdot x}\rightarrow\frac{4}{x}$ c. $\frac{x+1}{3(x+1)}\rightarrow\frac{1}{3}$ d. $\frac{(x+5)(x-5)}{x+5}\rightarrow x-5$
18.	a. $\frac{(x+6)(x+3)}{(x+4)(x+3)}\rightarrow\frac{x+6}{x+4}$ b. $\frac{4(x+5)}{2x(x+5)}\rightarrow\frac{2}{x}$

19.	a. $\frac{x+3}{x+3}$ b. $\frac{2}{2}$ and $\frac{x+5}{x+5}$ or $\frac{2(x+5)}{2(x+5)}$
20.	a. $\frac{(x-6)(x+1)}{(x-6)(x-2)}\rightarrow\frac{x+1}{x-2}$ b. $\frac{(x+10)(x-10)}{(x+4)(x-10)}\rightarrow\frac{x+10}{x+4}$
21.	$\frac{(x+6)(x-3)}{-3x(x-3)}\rightarrow-\frac{x+6}{3x}$ or $\frac{-x-6}{3x}$
22.	$\frac{4x(x+3)}{6x(x-2)}\rightarrow\frac{2(x+3)}{3(x-2)}$
23.	$\frac{(x+5)(x+2)}{(x+5)(x-1)}\rightarrow\frac{x+2}{x-1}$
24.	a. −1 b. −1
25.	$-(x+9)$ or $-x - 9$
26.	$-(2x-5)$ or $-2x + 5$ or $5 - 2x$
27.	a. $-(x - 4)$ b. $-x + 6$; $-(x - 6)$ c. $-2x + 1$; $-(2x - 1)$
28.	a. $1 - x$ or $-x + 1$ b. $7 - p$ or $-p + 7$ c. $y - x$ or $-x + y$
29.	a. $-x - 5$ b. $x - y$ c. $4-x^2$
30.	$-1\rightarrow\frac{x-4}{4-x}=-1$
31.	$-\frac{x}{3}$
32.	a. −1 b. −1 c. −1 d. −1
33.	a. 1 b. −1 c. $-\frac{1}{2}$ d. $-(y-4)$
34.	a. $\frac{(x+7)(x-2)}{(2+x)(2-x)}\rightarrow-\frac{x+7}{x+2}$ b. $\frac{6x(6-x)}{(x-6)(x-6)}\rightarrow-\frac{6x}{x-6}$ or $\frac{6x}{6-x}$
35.	$\frac{(10-x)(8+x)}{2x(x-3)(x-10)}\rightarrow-\frac{8+x}{2x(x-3)}$

