

# SUMMIT MATH

Learn at your **OWN** pace.

# ALGEBRA 2

second edition

# 5 RATES: MOTION, WORK, & INTEREST

# ***CONTENTS***

Section 1	<b><i>INTRODUCTION TO RATES SCENARIOS</i></b> .....	<b>3</b>
Section 2	<b><i>RATES AND THEIR RECIPROCAL</i></b> .....	<b>7</b>
Section 3	<b><i>SCENARIOS INVOLVING WORK RATES AND COMBINED WORK</i></b> .....	<b>10</b>
Section 4	<b><i>SCENARIOS INVOLVING MOTION</i></b> .....	<b>17</b>
Section 5	<b><i>CALCULATING AVERAGE SPEED</i></b> .....	<b>25</b>
Section 6	<b><i>SCENARIOS INVOLVING INTEREST RATES</i></b> .....	<b>29</b>
Section 7	<b><i>CUMULATIVE REVIEW</i></b> .....	<b>35</b>
Section 8	<b><i>ANSWER KEY</i></b> .....	<b>39</b>
	<b><i>HOMEWORK &amp; EXTRA PRACTICE SCENARIOS</i></b> .....	<b>43</b>

*Section 2*  
***RATES AND THEIR  
RECIPROCAL***

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GUIDED DISCOVERY SCENARIOS

The next group of scenarios will lead you to compare a rate and the reciprocal of that rate in order to see that both of these forms of a rate can be useful.

7. If a rate is measured in miles per hour, it may sometimes be helpful to express the rate as a reciprocal.

As a reciprocal, the units of this rate are \_\_\_\_\_.

8. For example, the reciprocal of 10 miles per hour would be 1 hour per 10 miles. If you convert this to a unit rate, it becomes 0.1 hour per mile. Write the reciprocal of each rate shown below. Convert the reciprocal rate into a unit rate.

a. 8 people per pizza

b. 0.25 inch per minute

9. A runner travels a distance of 3.6 miles in 24 minutes.

a. You can calculate that the runner moved at a rate of \_\_\_\_ miles per minute.

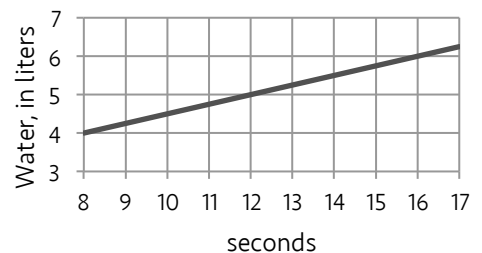
b. However, it might be more useful to state that the runner maintained a running pace of \_\_\_\_ minutes per mile, or \_\_\_\_ minutes and \_\_\_\_ seconds per mile. Distance runners are often interested in the time that it takes to travel each mile over the course of a long run.

c. What was the runner's speed, measured in miles per hour?

10. Ajay lives 24 miles from school and his car can travel 400 miles on 12 gallons of gas. How many gallons of gas will his car consume when he drives to school and back home again?

11. Consider the graph shown.

a. Describe the rate shown by the data in the graph.



b. Express the reciprocal of this rate, using proper units.

*Section 3*  
***SCENARIOS INVOLVING  
WORK RATES AND  
COMBINED WORK***

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12. In an effort to help out at a local fundraiser, you offer to wrap gifts for \$2 each. You can wrap 1 gift in 4 minutes. What portion of a gift can you wrap every minute? (This is your wrapping rate, measured in gifts per minute.)
13. In the previous scenario, how much money would you earn for the fundraiser in 1 hour?
14. When it snows outside, you like to clear the snow off of your neighbor's driveway. It takes you 40 minutes to clear all of the snow.
- What is the rate at which you work, measured in driveways per minute?
  - How much of the driveway do you clear off in 10 minutes?
  - How much of the driveway do you clear off in  $M$  minutes?
15. Suppose it takes a pump 6 minutes to fill a gas tank.
- How much of the tank is filled after 1 minute?
  - How much of the tank is filled after  $M$  minutes?
  - What is the rate at which the pump fills the tank, measured in tanks per minute?

16. Caleb plants 60 blueberry bushes in 8 hours. Gerry plants 50 blueberry bushes in 4 hours.
- What is Caleb's work rate?
  - What is Gerry's work rate?
  - When Gerry and Caleb work together, how many bushes can they plant in 1 hour?
  - If they work together, how long will it take them to plant 240 blueberry bushes?

17. If you can paint a room in 2 hours and your friend can also paint the entire room in 2 hours, estimate how long it will take to paint the room if you work together at the same time.

18. Read the scenario below and make a quick guess. Do not try to set up and solve an equation. The goal here is to see if you can get close just by guessing.

One caterpillar can eat an entire leaf in 12 hours. Another caterpillar can eat a leaf in 8 hours. Estimate the amount of time it would take for both caterpillars to eat the same leaf if they start at opposite ends.

19. Guess the missing information in the scenario below. Do not set up and solve an equation.

One beaver can build a dam in 4 days. Another beaver can build a dam in 6 days. If they work together at the same time, they can build a dam in \_\_\_\_\_ days.

*Section 5*  
***CALCULATING AVERAGE  
SPEED***

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44. If you ride in a plane for 3 hours and the flight takes you a distance of 1800 miles, you have enough information to calculate the average speed of the plane during this trip. Your average speed would be \_\_\_\_\_ miles per hour.

Since the plane speeds up at the start and slows down at the end, it does not fly at one speed for the entire trip, but you can still calculate the average speed to get an estimate of the speed of the plane.

45. A migrating flock of geese can fly 1000 miles in 25 hours. Although the speed of the geese may change as they fly, what is the average flying speed of the geese during their migration?
46. What information do you need in order to calculate the average speed of an object in motion?
47. Diana runs around town for 2.5 hours and she maintains a speed of 8 miles per hour. She then walks slowly for one half of an hour, moving at a rate of 2 miles per hour.
- A quick mental calculation reveals that the average of 8 miles per hour and 2 miles per hour is 5 miles per hour. Why is it unreasonable to state that Diana's average speed for the 3 hours that she was moving was 5 miles per hour?
  - What was the total distance that Diana traveled during her 3 hours of exercise?
  - What was Diana's average speed over the course of her 3 hours of motion?

*Section 6*  
***SCENARIOS INVOLVING  
INTEREST RATES***

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52. If you put your money in an account that earns interest, the value of your money increases over time. For example, suppose you put \$1,000 in a savings account that has a 2% interest rate. If you leave your money in the account for one year, its value will increase by 2%.
- After one year, your account is now worth \$\_\_\_\_\_.
  - The value of your account increases by \$\_\_\_\_\_ during the first year.
  - How much does the value of your account increase during the second year?
53. If someone has \$1,000 in a savings account and they add money at a rate of \$50 per day, how much money will be in the account after 10 days of increases?
54. If someone has \$1,000 in a savings account and they add money at a rate of 5% per day, how much money will be in the account after 10 days of increases?
55. Helen has \$10,000 in her bank account. Sam needs to borrow money to buy a car. He asks Helen for \$5,000. Helen gives Sam \$5,000, but she wants to earn some money in the process so she makes him agree to pay her an extra \$10 per month until he has paid back all that he owes.
- If it takes Sam two full years to pay back what he borrowed, how much money does Helen earn for loaning Sam \$5,000?
  - By what percent did Helen increase the value of her \$5,000 loan by making Sam pay extra?

# Section 8

# ANSWER KEY

1.	300 questions per day
2.	6 gallons/day (180 gal. over 30 days)
3.	Yes, she will use approx. $2\frac{1}{3}$ GB of data.
4.	a. Fred: 24 min; Cathy: 38 min b. Fred (6 per min.) > Cathy (5 per min.)
5.	a. $4000 + 120(12) = 5,440$ gallons b. $66\frac{2}{3}$ minutes $\rightarrow$ 1 hr, 6 min, 40 sec c. $9,000 - 120M$
6.	a. solve: $4000 + 120M = 9000 - 120M$ ; $M = 20\frac{5}{6}$ minutes - 20 min, 50 sec b. solve: $4000 + 120M = 2(9000 - 120M)$ ; $M = 38\frac{8}{9}$ minutes - approx. 8:39pm
7.	hours per mile
8.	a. $\frac{1}{8}$ of a pizza per person b. 4 minutes per inch
9.	a. 0.15 mi./min    b. $6\frac{2}{3}$ min/mi. or 6 min, 40 sec. per mile    c. 9 mph
10.	1.44 gallons (48 miles at $33\frac{1}{3}$ mi per gal.)
11.	a. 0.25 liters of water per second b. 4 seconds per liter of water
12.	$\frac{1}{4}$ of a gift per minute
13.	1 gift every 4 minutes for is a total of 15 gifts in 1 hour, so you earn \$30.
14.	a. $\frac{1}{40}$ driveway per minute    b. $\frac{10}{40} = \frac{1}{4}$ c. $\frac{1}{40}M$ or $\frac{M}{40}$
15.	a. $\frac{1}{6}$ b. $\frac{m}{6}$ c. $\frac{1}{6}$ tank per minute
16.	a. 7.5 bushes per hr    b. 12.5 bushes/hr c. 20 bushes d. Their combined work rate is 20 bushes

	per hour, so it will take them 12 hours.
17.	1 hour
18.	Your guess should be between 4 and 6 hours. The exact time is 4.8 hours...you'll learn how to find this later.
19.	Your guess should be between 2 and 3 days. The exact time is 2.4 days...you'll learn how to find this later.
20.	a. $\frac{3}{12}t + \frac{4}{12}t = 1 \rightarrow \frac{7}{12}t = 1 \rightarrow t = \frac{12}{7}$ or $1\frac{5}{7}$ b. $\frac{8}{40}h + \frac{5}{40}h = 1 \rightarrow \frac{13}{40}h = 1$ $\rightarrow h = \frac{40}{13}$ or $3\frac{1}{13}$
21.	a. 30 minutes    b. $17\frac{1}{7}$ min solve $\frac{1}{40}t + \frac{1}{30}t = 1$ ; $H = \frac{120}{7}$ ( $17\frac{1}{7}$ min)
22.	solve $\frac{1}{4}H + \frac{1}{5}H = 1$ ; $H = \frac{20}{9}$ ( $2\frac{2}{9}$ hrs)
23.	a. Carl = $\frac{1}{12}$ , Ryan = $\frac{1}{10}$ b. $\frac{11}{30}$ c. $\frac{11}{20}$ d. $\frac{11m}{60}$ e. $5\frac{5}{11}$ min (5 min, 27 sec)
24.	$\frac{2}{11}$ of a hour
25.	a. 16    b. 54 minutes
26.	a. Estimates close to 20 are accurate. b. $\frac{1}{30}(12) + \frac{1}{B}(12) = 1 \rightarrow 20$ minutes
27.	Bette builds 1 bike every 3.5 hours, or 11.4 per week. Sara builds 1 bike every 5 hours, or 8 per week. In 1 week, Bette builds about 3.4 more bicycles than Sara.
28.	Equation: $\frac{1}{x}(3) + \frac{2}{x}(3) = 1$ ; 4.5 hours
29.	27 miles
30.	distance $\div$ time
31.	$600 \div 240$