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ALGEBRA 2

second edition

7 EXPONENTIAL FUNCTIONS

CONTENTS

Section 1

INTRODUCTION TO EXPONENTIAL PATTERNS

1. On the first day of the school year, you walk into a class and your teacher says, "We are going to take a different approach to studying this year. To start with, during Week 1, you will only study your notes for 2 seconds. During Week 2, you will have more to remember so you will double your studying time and spend 4 seconds studying. During Week 3, you will double your amount of studying again, and you will continue this pattern until the end of the year."

   a. How much time will you spend studying during the 8th week of the class?

   b. If the midterm exam comes at the end of the 15th week, how much time will you set aside that week to study for the exam?

   c. How long will you spend studying your notes during the week before you take the final exam if the class ends after 30 weeks?

   d. How long will you spend studying your notes during the *n*th week, if "*n*" represents any week you would like to consider?

2. In the previous scenario, the amount of time that you spend studying increases by ____% every week.

3. In 2014, to raise awareness of the disease amyotrophic lateral sclerosis (ALS), a fundraiser was developed that became known as the ALS ice bucket challenge. It became popular around the world and simple mathematics can explain why this happened. The challenge starts with one person, who dumps ice water over his or her head and then challenges someone they know to do the same within 24 hours or donate money toward funding ALS research. Suppose everyone who completes the ice bucket challenge then challenges 3 other people. Assume that all challenges are accepted, no person is challenged more than once, and a new round of ice bucket challenges occurs every day.

   a. A golfer named Chris Kennedy allegedly started the ALS ice bucket challenge. One day after Chris, 3 people did the ice bucket challenge. Two days after Chris, how many people dumped ice water over their heads?

   b. How many people did the ice bucket challenge 4 days after Chris?

   c. How many people dumped ice water over their heads 10 days after Chris?

   d. How many people did the ice bucket challenge *n* days after Chris?

4. In the previous scenario, the number of people who did the challenge increased by ____% every day.

Section 2
EXPONENTIAL SEQUENCES

13. Repeated Multiplication.

a. Use a calculator to multiply 1.5 × 1.5 × 1.5 × .... If there are 26 of those 1.5's in this string of multiplied values, what could you type into the calculator to find the result quickly?

b. Repeated multiplication is more quickly calculated with __________________.

c. What is the 18th term in the sequence 1.5, 2.25, 3.375, 5.0625, ...? Round to the tenth.

d. What is the *n*th term in the sequence 1.5, 2.25, 3.375, 5.0625, ...?

14. The *n*th term in a sequence of numbers is defined by the expression $10(4)^n$. Write the first 3 terms of the sequence (Let n = 1, 2, and 3).

15. Suppose the *n*th term in a sequence is $7(2.5)^n$. Write the first 3 terms of the sequence.

16. The expression $10 \cdot 2^n$ looks like it describes each term in the sequence in the table below.

1st term (n=1)	2nd term (n=2)	3rd term (n=3)	4th term (n=4)
10	20	40	80

a. See if $10 \cdot 2^n$ gives the terms in the sequence above by replacing n with 1, 2, 3, and 4.

b. Change the 10 in $10 \cdot 2^n$ to a different number to make it fit the sequence.

17. Fill in the blank. The *n*th term in the sequence below is $\_\_\_\_ \cdot 5^n$.

1st term (n=1)	2nd term (n=2)	3rd term (n=3)	4th term (n=4)	5th term (n=5)
20	100	500	2500	12500

18. Consider the sequence 6, 12, 24, 48, 96, . . .

a. What is the next term in the sequence?

b. What is the *n*th term in the sequence?

Section 5
EXPONENTIAL FUNCTIONS

It should now be clear to you that exponential growth can be represented, or modeled, by the following function: $y = A(B)^T$. The letters can be changed to any letters that you would like. The important thing is to know what they represent.

51. The number of rabbits in a population is modeled by the function $P = 600(1.03)^d$, where *P* is the population when it is measured *d* days after April 1st.

a. Is the population increasing or decreasing every day? By what percent?

b. What is population on April 1st?

c. Describe two ways to determine the population on April 3rd.

52. The population of a city in Ohio is modeled by the function $P = 28{,}000(0.98)^y$, where *P* is the population *y* years after 2010.

a. Is the population increasing or decreasing every year? By what percent?

b. What was the population of the city in 2013?

c. Determine the population in 2010.

53. Write an exponential function to model each scenario.

a. An initial value of 72 increases 20% every year.

b. An initial value of 200 decreases 60% every year.

54. ★Clara purchased a used car for $21,000.

a. If the resale value of the car has been decreasing by 15% every year, what was the resale value of the car one year earlier? Round to the nearest dollar.

b. What was the resale value of the car 4 years ago? Round to the nearest dollar.

c. What expression represents the resale value *N* years ago?

Section 8

WRITING AN EXPONENTIAL FUNCTION, GIVEN 2 POINTS

An exponential function has the form $y = A(B)^x$, where *A* and *B* have numerical values. There are two values that you need to know (*A* and *B*) in order to write an exponential function.

84. Suppose there is an exponential function that contains the points $(1, 3)$ and $(4, 24)$. An exponential function in its general form has the structure $y = A(B)^x$.

    a. Replace *x* and *y* with 1 and 3, respectively, in the equation $y = A(B)^x$.

    b. Now replace *x* and *y* with 4 and 24, respectively, in the equation $y = A(B)^x$.

85. At this point, you have two equations that contain the same pair of variables. As with linear systems of equations, you can solve this system of equations to determine the values of *A* and *B*. You may be unable to solve this system, but take a moment to see if you can figure out how to do this. If you get stuck, move on to the next scenario.

86. When you substitute the points $(1, 3)$ and $(4, 24)$ into the equation $y = A(B)^x$, it makes two equations, $3 = A(B)^1$ and $24 = A(B)^4$.

    a. In order to use the Substitution Method for solving a system of equations, isolate *A* in the equation $3 = A(B)^1$.

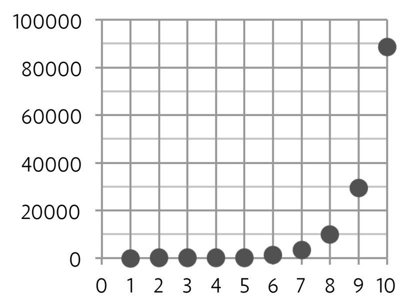
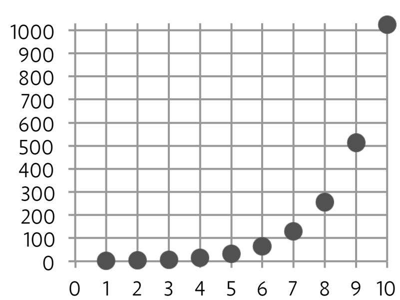
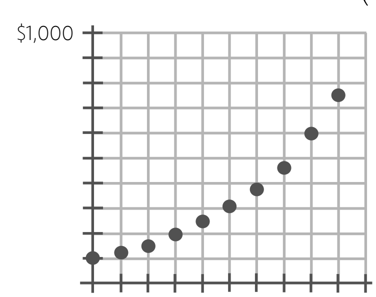
    b. In the other equation, $24 = A(B)^4$, make a substitution by replacing *A* with $\frac{3}{B^1}$.

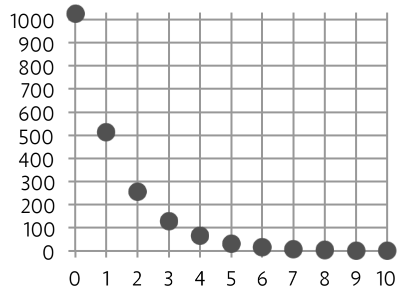
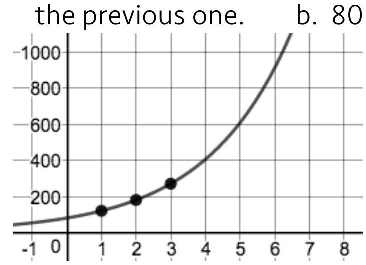
    c. Now that this equation only contains one variable, solve for *B*.

    d. In either of the original two equations, replace *B* with 2 and solve for *A*.

87. An exponential function has the form $y = A(B)^x$. In the previous scenario, you found the values of *A* and *B* for the exponential function that contains the points $(1, 3)$ and $(4, 24)$. Now that you have these values, write the function.

Section 12
ANSWER KEY

1.	a. 256 sec. (≈ 4.3 minutes) b. 2^{15} or 32,768 sec. (≈ 9.1 hours) c. 2^{30} ≈ 1 billion sec. (≈ 34 years) d. 2^n
2.	100
3.	a. $3 \cdot 3 = 9$ people b. 3^4 or 81 people c. 3^{10} or 59,049 people d. 3^n
4.	200
5.	a. $1 + 3^1 + 3^2 + 3^3 \rightarrow$ 40 people b. $1 + 3^1 + 3^2 + \ldots + 3^5 \rightarrow$ 364 people c. $1 + 3^1 + 3^2 + \ldots + 3^{10} \rightarrow$ 88,573 people
6.	
7.	This growth cannot continue, because 21 days after Chris begins, the number of participants is more than 15 billion, which is more than the global population.
8.	a. 2^n b. 3^n
9.	a. $45 \cdot 10$ b. multiplication c. $17 \cdot 10$ or 170 d. $10n$
10.	a. $64 \cdot 15$ b. $24 \cdot 15$ or 360 c. $15n$
11.	a. 2^{25} or 33,554,432 b. exponents c. 2^{20} or 1,048,576 d. 2^n
12.	
13.	a. $(1.5)^{26}$ b. exponents c. $(1.5)^{18}$ or 1,477.9 d. $(1.5)^n$
14.	40, 160, 640
	$10(4)^1 = 40,\ 10(4)^2 = 160,\ 10(4)^3 = 640$
15.	17.5, 43.75, 109.375
16.	a. $10 \cdot 2^n$ does not describe the sequence. b. $5 \cdot 2^n$
17.	$4 \cdot 5^n$
18.	a. 192 b. $3 \cdot 2^n$
19.	a. $5 \cdot 2^{20}$ or 5,242,880 b. $5 \cdot 2^n$
20.	$2 \cdot 3^n$
21.	$7 \cdot 5^n$
22.	Choose two consecutive numbers. Divide the second number by the first number. The result is 5.
23.	a. 45.5625 b. $4(1.5)^n$
24.	$2(4.5)^n$
25.	$8(2.1)^n$
26.	The numbers in the sequence are formed by repeatedly multiplying by a fixed value. If you call this value N, each number is N times the previous number.
27.	a. 0.5A b. 0.03B c. 2.75C d. 0.004D
28.	a. 120 b. 80 c. 108 d. 52
29.	a. $A + 0.5A$ b. $B - 0.25B$ c. $C + 0.05C$ d. $D - 0.15D$
30.	a. $1.5A$ b. $0.75B$ c. $1.05C$ d. $0.85D$
31.	a. 1.35X b. 0.81X c. 2X d. 0 e. 170 f. 75
32.	a. 125 b. ≈476.8 c. $100(1.25)^n$
33.	a. \$125 b. ≈\$476.84 c. $100(1.25)^n$ 
34.	a. ≈16,105 b. $10{,}000(1.1)^n$

35.	a. ≈\$2,687.83 b. $2{,}000(1.03)^n$
36.	a. 32 b. $1{,}024(0.5)^n$ 
37.	a. 32mg b. $1{,}024(0.5)^n$
38.	60%
39.	$(0.8)^4 = 0.4096 \approx 41\% \rightarrow$ about 41% remains
40.	$(0.7)^n \rightarrow$ Convert the decimal to a percent.
41.	a. ≈478 b. $1{,}000(0.9)^n$
42.	a. ≈\$23,620 b. After 4 years of 10% decreases, it would be worth 90% of 90% of 90% of 90% of its value, which is $(0.9)^4$ or 65.61% of its original value. This is a decrease of 34.39%. c. $36{,}000(0.9)^n$
43.	\$2,100 decreased by 5% is \$1,995. \$1,995 increased by 5% is \$2,094.75, which shows that \$1,995 was not last year's value. To find last year's value, define a variable, *x*. Solve the equation 1.05*x* = 2,100 to find what last year's value was before it rose 5% to become \$2,100.
44.	2,000 gallons. You cannot increase 18,000 by 10% to get the original amount. Instead, you must decrease the original by 10% to make it become 18,000. Solve the equation 0.9*x* = 18,000. Since *x* = 20,000, you need to add 2,000 more gallons.
45.	$120(0.9)^n$
46.	a. $B = N(1.08)^5$ b. $P = C(0.85)^8$
47.	*A* is 1.27
48.	*B* is 0.65
49.	a. $M = D\left(1+\frac{R}{100}\right)^4$ b. $V = D\left(1-\frac{R}{100}\right)^7$
50.	a. $A = M\left(1-\frac{R}{100}\right)^D$ b. $T = P\left(1+\frac{R}{100}\right)^W$
51.	a. increasing by 3%
	b. 600 (let *d* = 0 in the function) c. OPTION 1: Increase 600 by 3% to get 618. Increase 618 by 3% to get 636.54. OPTION 2: Let *d* = 2 in the function.
52.	a. decreasing by 2% b. 26,353 (let *y* = 3 in the function) c. 28,000 (let *y* = 0 in the function)
53.	a. $V = 72(1.20)^x$ b. $V = 200(0.40)^x$
54.	a. ≈\$24,706 b. ≈\$40,229 c. $\frac{21{,}000}{(0.85)^N}$ or $21{,}000(0.85)^{-N}$
55.	a. ≈108,688 b. $\frac{120{,}000}{(1.02)^N}$ or $120{,}000(1.02)^{-N}$
56.	a. $V = 20{,}000(1.076)^y$ b. the painting (\$100,207 vs. \$99,577)
57.	a. \$200 b. \$190 c. approx. \$113.76 (after the 11th payment, they owe \$2,275.20 and 5% of that is \$113.76) d. After the 35th payment, the remaining balance is approx. \$664.33 so their final payment will be that full amount.
58.	a. the difference between consecutive numbers is the always the same b. 3*n* + 5
59.	Each number in the sequence is the previous number multiplied by the same amount. In this sequence each term is the previous term multiplied by 1.2.
60.	a. Each *y*-value is 50% higher than the previous one. Also, each *y*-value is 1.5 times the previous one. b. 80 and 405 c.  d. $(0,80)$
61.	$y = 80(1.5)^x$
62.	$\frac{9.\overline{09}}{13.31}$
63.	$y = 10(1.1)^x$
64.	a. $\frac{56.25}{23.04}$ b. $\frac{51.2}{156.25}$
65.	b. $y = 45(0.8)^x$ c. $y = 64(1.25)^x$
66.	A: $y = 45(0.8)^x$ B: $y = 64(1.25)^x$

