SUMMIT MATH

*Learn at your **OWN** pace.*

ALGEBRA 2

second edition

8

THE PYTHAGOREAN THEOREM & SPECIAL RIGHT TRIANGLES

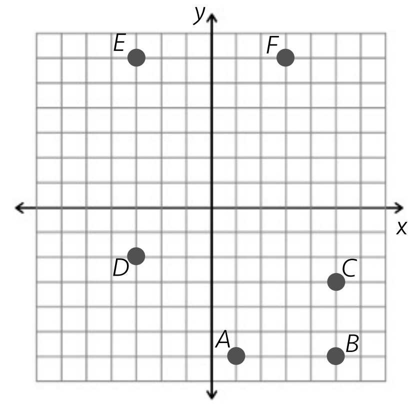
CONTENTS

Section 2

THE DISTANCE BETWEEN TWO POINTS

Use the graph shown to the right for the next 4 scenarios.

16. If the grid lines are each a distance of 1 unit apart, what is the distance between each pair of points shown?



a. *A* and *B*   b. *B* and *C*   c. *A* and *C*

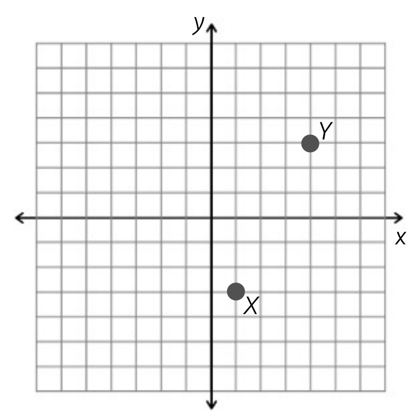
17. If it is difficult to find the distance between *A* and *C*, it may be helpful to see the points *A*, *B*, and *C* as the corners of a right triangle. How does this help you find the distance between *A* and *C*?

18. In the graph above, what is the distance between each pair of points shown?

a. *D* and *E*   b. *E* and *F*   c. *D* and *F*

19. In the previous scenario, since you cannot use the grid lines to count the distance between points *D* and *F*, how can you determine the distance between those two points?

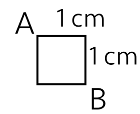
20. Another group of points is shown to the right. What is the distance between the points *X* and *Y*?



21. Use a calculator to write the previous distance in its rounded form, to the nearest tenth.

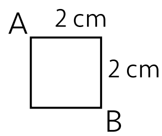
Section 4
DIVIDING A SQUARE TO MAKE A SPECIAL RIGHT TRIANGLE

41. Use the Pythagorean Theorem to find the distance between point A and point B (the opposite corners of the square). Write your result in simplified radical form.

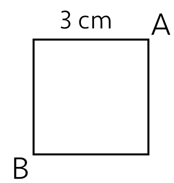


42. Use the Pythagorean Theorem to find the distance between points A and B in each square. Write each result in simplified radical form.

a.

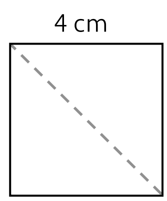


b.

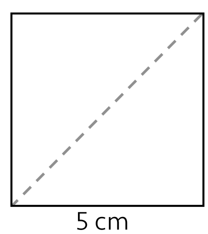


43. Do you notice any patterns yet? Find the length of the diagonal line drawn in each square below.

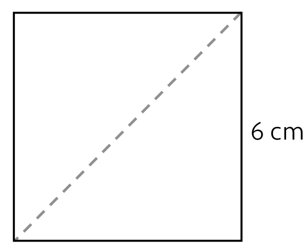
a.



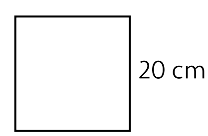
b.



c.



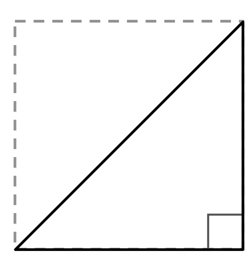
44. Look at the relationship between the side lengths and the diagonal length for each of the previous six squares. In the square to the right, what is the distance between opposite corners?



45. Another special thing about squares is their angles.

a. How large is each angle in a square?

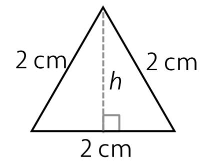
b. If you draw a line to connect opposite corners in a square, it splits the square into two congruent ________________________ triangles. Fill in the two unmarked angles in the ________________________ triangle shown to the right.



Section 7

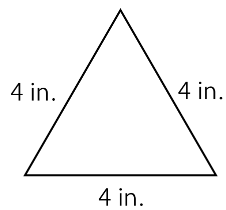
DIVIDING AN EQUILATERAL TRIANGLE TO MAKE A SPECIAL RIGHT TRIANGLE

79. Use the Pythagorean Theorem to find the **height** of the triangle. Write the height in simplified radical form.

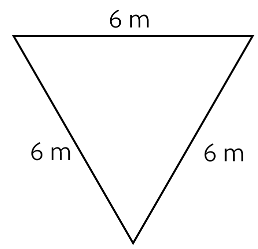


80. Use the Pythagorean Theorem to find the **height** of each triangle. Write your answers in simplified radical form.

a.

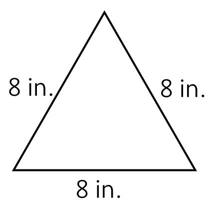


b.

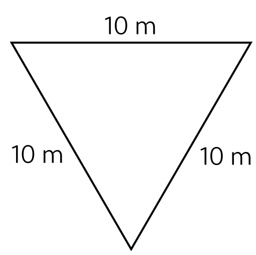


81. Find the **height** of each triangle shown below and try to find a relationship between the height and the triangle's side lengths.

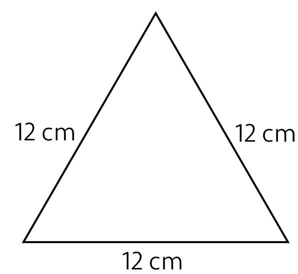
a.



b.

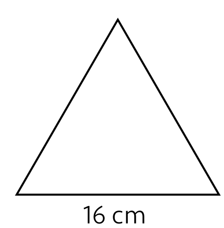


c.



82. Try to find a pattern in the previous scenarios by comparing the height and the side lengths of each equilateral triangle. If you see this pattern, the next question can be answered without using the Pythagorean Theorem.

What is the height of the equilateral triangle shown?

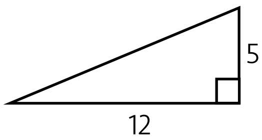


Section 9

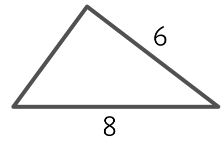
RIGHT TRIANGLE SCENARIOS

113. It is important to remember that most right triangles are not 45-45-90 or 30-60-90 triangles. To help you remember this, find the missing length of each triangle below.

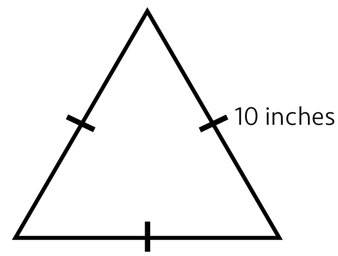
a.



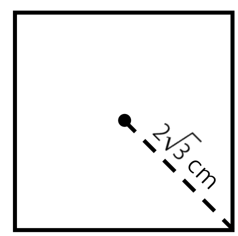
b.



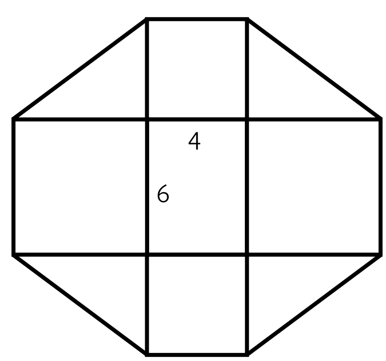
114. Determine the area of the triangle.



115. Calculate the area and perimeter of the square. The dot marks the center of the square.



116. Determine the outside perimeter of the figure if the inside of the figure contains 4 squares, 4 triangles and 1 rectangle.

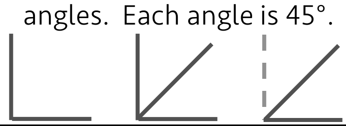
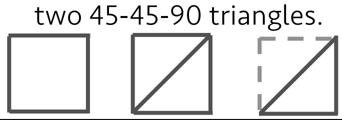


Section 11

ANSWER KEY

1.	a. $a^2+b^2=c^2$ b. It shows the relationship between the side lengths of a right triangle. If you know the lengths of 2 sides of a right triangle, you can use the P.T. to find the length of the 3rd side.
2.	c; hypotenuse; legs a^2+b^2 is the same as b^2+a^2 due to the commutative property of addition
3.	a. 9 b. 5
4.	a. $8x^2$ b. $10\sqrt{21}$ c. $4\sqrt{49} \to 4 \cdot 7 \to 28$
5.	a. $2\sqrt{2}$ b. $2\sqrt{3}$
6.	a. 10 b. $\sqrt{10}$ c. $2\sqrt{10}$ d. 6
7.	a. $5\sqrt{4} \to 10$ b. $30\sqrt{2}$
8.	a. $x = 2\sqrt{5}$ b. $x = 13$ c. $x = 2$
9.	a. No, they have different solutions. b. No
10.	a. A and C b. D and E c. E and G
11.	hypotenuse
12.	a. 5 b. $2\sqrt{5}$ c. $5\sqrt{2}$
13.	$\sqrt{a^2+b^2}$
14.	a. 2 b. 10 c. $\sqrt{3}$
15.	It is not possible to calculate the unmarked side. The Pythagorean Theorem only applies to right triangles and the triangle shown does not contain a right angle.
16.	a. 4 b. 3 c. 5
17.	AC is the hypotenuse of a right triangle with side lengths 3 and 4
18.	a. 8 b. 6 c. 10
19.	Connect the dots to form a right triangle. Measure the legs of the right triangle and find the length of the hypotenuse with the Pythagorean Theorem.
20.	$\sqrt{3^2+6^2} \to \sqrt{45} \to 3\sqrt{5}$
21.	6.7 units
22.	a. $2\sqrt{5}$ b. $3\sqrt{2}$ c. 10
23.	hypotenuse; Pythagorean
24.	a. a=4; b=8; distance= $4\sqrt{5} \approx 8.9$ b. a=6; b=8; distance=10

25.	a. a=108; b=54; distance= $\sqrt{14{,}580} \approx 120.7$
26.	Subtraction. If you subtract two numbers, the result is the distance between those two numbers.
27.	option 1: $y-(-10)$ or $y + 10$ option 2: $-10 - y$
28.	a. $\sqrt{(A-5)^2+(3-B)^2}$ or $\sqrt{(5-A)^2+(B-3)^2}$ b. $\sqrt{(-3-D)^2+(C+2)^2}$ or $\sqrt{(D+3)^2+(-2-C)^2}$
29.	$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ or $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
30.	The triangle is not a right triangle, so you do not have enough information to calculate the length of side GH.
31.	$\sqrt{4^2+(-3)^2} \to \sqrt{16+9} \to \sqrt{25} \to 5$
32.	(7, 1) and (3, 4)
33.	a. 5 b. $4\sqrt{2}$
34.	a. (5, −5) and (2, −1) b. (−4, 2) and (−8, 6)
35.	a. 5 b. 13
36.	a. $2\sqrt{5} \approx 4.5$ b. $5\sqrt{5} \approx 11.2$
37.	side lengths: 5 a. 20 b. 25
38.	width: $2\sqrt{2}$ length: $4\sqrt{2}$ Area: 16
39.	sides: 5, 10, $5\sqrt{5}$ a. $15 + 5\sqrt{5}$ b. 25
40.	Two sides have opposite reciprocal slopes, which makes them perpendicular.
41.	$\sqrt{2}$
42.	a. $2\sqrt{2}$ b. $3\sqrt{2}$
43.	a. $4\sqrt{2}$ b. $5\sqrt{2}$ c. $6\sqrt{2}$
44.	$20\sqrt{2}$ cm
45.	a. 90° b. right isosceles; 45° and 45°
46.	a. $v = 4$; $w = 4\sqrt{2}$ b. $v = w = 11$
47.	a. $v = 0.5$; $w = 0.5\sqrt{2}$ b. $w = 13\sqrt{2}$
48.	a. $x\sqrt{2}$ b. $7\sqrt{2}$ c. 15
49.	44
50.	Isosceles Right Triangle
51.	a. leg: x; hyp: $x\sqrt{2}$ b. leg: x; hyp: $x\sqrt{2}$ c. both legs: x
52.	20 inches

53.	a. legs: 2 b. leg: 15; hyp: $15\sqrt{2}$ c. hyp: $\sqrt{2}$
54.	a. $3\sqrt{2}$; 6 b. $4\sqrt{2}$; 8
55.	$10\sqrt{2}\sqrt{2} \rightarrow 20$
56.	a. 14 b. 2 c. 12
57.	$3\sqrt{10}$
58.	a. leg: x; hyp: $x\sqrt{2}$ b. leg: x; hyp: $x\sqrt{2}$ c. both legs: x
59.	both legs: $2\sqrt{2}$
60.	Divide both sides by $\sqrt{2}$.
61.	Rationalize the denominator by multiplying the fraction by $\frac{\sqrt{2}}{\sqrt{2}}$ to get $\frac{4\sqrt{2}}{2}$, or $2\sqrt{2}$.
62.	Multiply $2\sqrt{2}$ by $\sqrt{2}$ to confirm that the result is 4 (the length of the hypotenuse).
63.	a. $x = 6\sqrt{2}$ b. $x = \frac{\sqrt{2}}{2}$
64.	a. $\sqrt{2}$ b. $7\sqrt{2}$ c. $\frac{5\sqrt{2}}{2}$
65.	a. $4^2 + \left(4\sqrt{2}\right)^2 = c^2 \rightarrow c^2 = 48 \rightarrow c = 4\sqrt{3}$ in b. not possible; the hypotenuse must be the longest side. The missing side would need to be 0 for the hypotenuse to have the same length as the leg.
66.	a. leg: x; hyp: $x\sqrt{2}$ b. leg: x; hyp: $x\sqrt{2}$ c. both legs: x
67.	There is not enough information given. These are not 45-45-90 triangles.
68.	This is not a 45-45-90 triangle. There is not enough information given.
69.	Draw a right angle and divide it into 2 equal angles. Each angle is 45°. 
70.	Draw a square. Connect opposite corners with a diagonal. The square is now split into two 45-45-90 triangles. 
71.	a. 15 b. $4\sqrt{5} \approx 8.9$
72.	a. $\frac{9}{12} \rightarrow \frac{3}{4}$ b. $\frac{-4}{8} \rightarrow -\frac{1}{2}$
73.	a. 5 b. 6 c. 12 d. $5x$
74.	Each of the triangles has side lengths with a ratio of 3:4:5. This type of triangle is commonly called a 3-4-5 right triangle.

75.	Draw a square and split it in half along its diagonal to make two 45-45-90 triangles.
76.	a. $10 + 10\sqrt{2}$ units b. Two sides have equal lengths. Those sides also have opposite reciprocal slopes. Thus, the figure is an Isosceles Right Triangle.
77.	a. Method 1: Use the perpendicular sides as the base and the height. Method 2: Draw a rectangle around the triangle, which also creates 3 smaller right triangles. Subtract the areas of the 3 right triangles from the area of the surrounding rectangle to get the area of the interior triangle. b. 25 square units
78.	There are two possible values. If B is the right angle, then x is the hypotenuse. Thus, $4^2 + 2^2 = x^2 \rightarrow x = 2\sqrt{5}$. If C is the right angle, then 4 is the hypotenuse. Thus, $2^2 + x^2 = 4^2 \rightarrow x = 2\sqrt{3}$.
79.	$\sqrt{3}$ cm
80.	a. $2\sqrt{3}$ in. b. $3\sqrt{3}$ m
81.	a. $4\sqrt{3}$ in. b. $5\sqrt{3}$ m c. $6\sqrt{3}$ cm
82.	$8\sqrt{3}$ cm
83.	Write the word "equilateral" in the blank. The height is $10\sqrt{3}$ cm.
84.	$21\sqrt{3}$ inches
85.	$180° \div 3 \rightarrow 60°$
86.	Write the word "right" in both of the blanks. The angles of the triangle are 30° and 60° .
87.	a. $9\sqrt{3}$ b. shorter: 7; longer: $7\sqrt{3}$ c. shorter: 12; longer: 24
88.	a. $v = 4\sqrt{3}$; $w = 8$ b. $v = 5\sqrt{3}$; $w = 5$
89.	a. $v = 22$; $w = 11$ b. $v = 0.5\sqrt{3}$; $w = 1$
90.	a. $\frac{x\sqrt{3}}{2}$ or $\frac{x}{2}\sqrt{3}$ b. $\frac{11\sqrt{3}}{2}$ or $5.5\sqrt{3}$ c. $\frac{5\sqrt{3}}{2}$ or $2.5\sqrt{3}$
91.	1 unit
92.	The 2 acute angles in a right triangle have a sum of 90°. a. 30° b. 53° c. 45°
93.	a. hypotenuse: $2x$; longer leg: $x\sqrt{3}$ b. shorter leg: x; longer leg: $x\sqrt{3}$